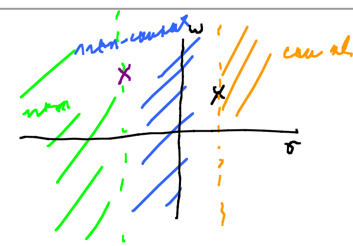
Note Title 9/19/2020

ENEE 610
10/22/20

Paper requirements, etc.
Causal vs non-causal=anti-causal
Richards' function via S & section for synthesis

Ladders for zeros of transmission
Constant R circuits &Vo(s)/Vi(s) design
exp(-At)
op-amp curve tracer
Chain matrix



An interesting paper:
Emil Cauer, Wolfgang Mathis, and Rainer Pauli, "Life and Work of Wilhelm Cauer (1900 – 1945)," Proceedings of the MTNS2000, Perpignan, France, June 19 - 23, 2000, 10 pages.

A good paper for ladder synthesis:
Alexander J. Casson ⋆ and Esther Rodriguez-Villegas, "A Review and Modern Approach to LC Ladder Synthesis,"

Richards' function $R_y(s)$ for $y(s)$ [R_y is PR if y is PR when k>0; and s^2-k^2 cancels to give δ[R]=δ[y]-1 If k is a zero of Ev[y(s)],that is Ev[y(k)={y(k)+y(-k)}/2=0; physically y_L is a load admittance. The result extends to complex k for Rek>0 and a similar Richards' function is useful for impedances and] :

$$R_y = \frac{k\,y(k)}{k\,y(k)}\left[\frac{1-\frac{s}{k}\cdot\frac{y(s)}{y(k)}}{\frac{y(s)}{y(k)}-s/k}\right];\quad S_{R_y}(s)=(B+A)^{-1}(B-A);\quad y(s) \Rightarrow i = Yv$$

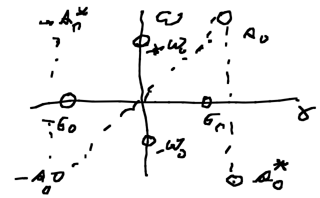
$$Av = Bi \Rightarrow B^{-1}Av = i;\quad Y = B^{-1}A$$

$$= (1+B^{-1}A)^{-1}B^{-1}B(1-B^{-1}A)$$
$$= (1+y)^{-1}(1-y)$$
$$= (1-R_y)/(1+R_y) = \frac{\left(1-\left(1-\frac{s}{k}\cdot\frac{y(s)}{y(k)}\right)/\left(\frac{y(s)}{y(k)}-s/k\right)\right)}{1+(\quad '' \quad / \quad '' \quad)} = \frac{\frac{y(s)}{y(k)}-\frac{s}{k}-1+\frac{s}{k}\cdot\frac{y(s)}{y(k)}}{\frac{y(s)}{y(k)}-\frac{s}{k}+1-\frac{s}{k}\cdot\frac{y(s)}{y(k)}}$$

$$= \frac{\frac{s}{k}\left(-1+\frac{y(s)}{y(k)}\right)+\left(-1+\frac{y(s)}{y(k)}\right)}{\frac{s}{k}\left(-1-\frac{y(s)}{y(k)}\right)+\left(1+\frac{y(s)}{y(k)}\right)} = \frac{(s+k)\left(-1+\frac{y(s)}{y(k)}\right)}{(-s+k)\left(1+y(s)/y(k)\right)}.$$

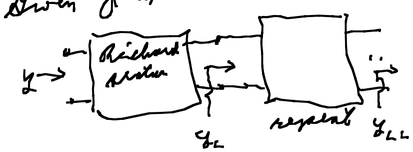
a zero & pole cancel (-s+k disappears & other term is $BR = S_{R_y}$

zeros of $Ev[y(s)] = \frac{y(s) + y(-s)}{2}$



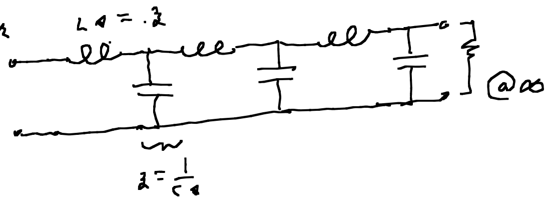
any real axis point is a zero for $Ev[y(s)]$ if $y(s)$ LPR

Given $y(s)$ LPR

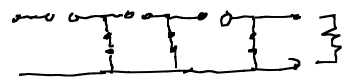


ends after $\delta[y_L]$ terms gives an ∞ number of lossless circuit

1st cauer
5 poles @ ∞



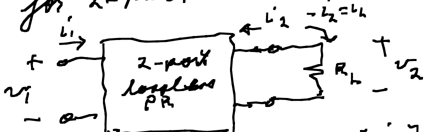
@ ∞



has 6 transmission zeros @ ∞.

$\frac{v_0}{v_i} = \frac{k}{s^6 + \dots + d_0}$ ← Hurwitz if load with R

Design for 2-port terminated in a resistor



$A_v(s) = \frac{v_2(s)}{v_1(s)}$

$-G_L v_2 = \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Rightarrow -G_L v_2 = y_{21} v_1 + y_{22} v_2$

$v_2 = \frac{y_{21}}{-G_L - y_{22}} v_1$

$\Rightarrow A_v = \frac{v_2}{v_1} = \frac{-y_{21}}{G_L + y_{22}}$ normalize $G_L = 1 = 1/R_L$

$$A_v = \frac{v_2}{v_1} = \frac{-y_{21}}{1+y_{22}} \quad ; \quad y_{22} \text{ LPR} \qquad y_{22} = \frac{n_{22}}{d_{22}} \ , \ y_{21} =$$

$$= \frac{-y_{21}}{1+n_{22}/d_{22}} = \frac{-y_{21}\,d_{22}}{n_{22}+d_{22}}$$

form from $A_v = \dfrac{N}{D} = \dfrac{N}{Ev[D]+Od[D]}$

$$= \frac{N/Od[D]}{1+\frac{Ev[D]}{Od[D]}} = \frac{-y_{21}}{1+y_{22}}$$

$$A_v = \frac{+k}{s^4+3s^3+5s^2+9s+4}$$

$$= \frac{k}{(s^4+5s^2+4)+(3s^3+9s)}$$

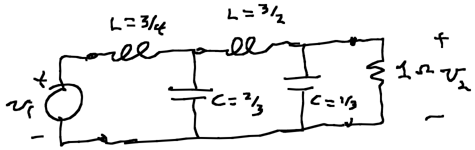
y_{22} choose $= \dfrac{s^4+5s^2+4}{3s^3+9s}$ LPR if A_v stable

$$-y_{21} = k/(3s^3+9s)$$

Synthesize y_{22} by 1st Cauer (as all 4 zeros of transmission are at ∞)

y_{22}: $3s^3+9s \,\big)\, s^4+5s^2+4 \,\big(\, \frac{1}{3}s$
$\quad s^4+3s^2$
$\quad 2s^2+4 \,\big)\, 3s^3+9s \,\big(\, \frac{3}{2}s$
$\quad 3s^3+6s$
$\quad 3s \,\big)\, 2s^2+4 \,\big(\, \frac{2}{3}s$
$\quad 2s^2$
$\quad 4 \,\big)\, 3s \,\big(\, \frac{3}{4}s$

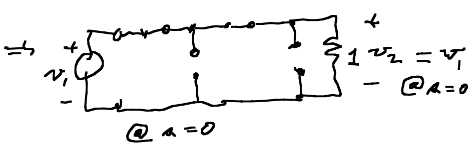
$$y_{22} = \frac{1}{3}s + \cfrac{1}{\frac{3}{2}s + \cfrac{1}{\frac{2}{3}s + \cfrac{1}{\frac{3}{4}s}}}$$



$$A_v = \frac{k}{s^4+3s^3+5s^2+9s+4}$$

$y_{21} = \dfrac{i_2}{v_1}\Big|_{v_2=0}$ ⇒ a short

to find k, $A_v(0) = \dfrac{k}{4}$ ⇒



$A_v(0) = 1 = k/4 \Rightarrow k = 4$

$$A_v = \frac{4}{s^4+3s^3+5s^2+9s+4} \quad ; \quad y_{21} = \underbrace{\frac{-4}{3s^3+9s}}_{\delta[K_{21}]=3}$$

$\delta[A_v] = 4$